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VARIATIONAL PRINCIPLE OF THERMOELECTROELASTICITY AND ITS APPLICATION TO THE PROBLEM OF VIBRATIONS OF A THIN-WALL MEMBER

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The dynamic behavior of thin-wall members manufactured from materials with the pyroelectric effect was studied. A variational formulation of the problem is used, and a variational principle is formulated that differs from the well-known one. Correct boundary-value problems describing the tension, compression, and bending of a thin-wall pyroelectric member are constructed using the variational principle and a number of hypotheses on the distribution of the components of physical fields along the width of the member.

Introduction. Structures and members manufactured from piezoactive materials are presently widely used in engineering. It is of interest to study their behavior under conditions of nonuniform temperature loading. Investigations and calculations were performed using the model of coupled thermoelectroelasticity [1], in which the system of equations is generally rather complex. The boundary-value problems of thermoelectroelasticity are based on the equations of thermopiezoelectricity formulated by R. D. Mindlin at the beginning of the 1960s of the 20th century. The mutual effect of thermal, electrical, and elastic fields was studied in [2–5]. A generalized formulation of the boundary-value problem of thermoelectroelasticity is given in [5]. In the solution of particular problems, a need arises to construct simpler models and equations.

In the present paper, we consider the system of equations and boundary conditions for the problem of vibrations of thin-wall members manufactured from materials with the piezo- and pyroelectric effect.

1. Formulation of the Problem. The problem of steady-state vibrations of a body which occupies volume V with boundary S is studied using the model of coupled linear thermoelectroelasticity. Assuming steady-state vibrations and omitting the time factor $\mathrm{e}^{-i\omega t}$, we write the basic equations in the form

$$\sigma_{ij,j} + F_i = -\omega^2 \rho u_i, \qquad D_{k,k} = 0, \qquad -i\omega T_0 \eta = -q_{i,i} + w. \tag{1}$$

The constitutive relations are [1]

$$\begin{gathered}\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - \gamma_{ij}\theta - e_{kij}E_k, \quad D_i = e_{ikl}\varepsilon_{kl} + g_j\theta + \epsilon_{ik}E_k, \quad \eta = \gamma_{ij}\varepsilon_{ij} + (c_\varepsilon/T_0)\theta + g_iE_i,\\ E_k = -\varphi_{,k}, \qquad \varepsilon_{ij} = (u_{i,j} + u_{j,i})/2, \qquad q_i = -k_{ij}\theta_{,j}.\end{gathered} \tag{2}$$

Here σ_{ij} are the stress-tensor components, D_i are the components of the electric-induction vector, η is the entropy, q_i are the components of the heat-flux vector, F_i are the components of the volume-force vector, w is the volume power of internal heat sources, u_j are the displacement-vector components, φ is the electric potential, θ is the temperature increment relative to the temperature in the natural state c_{ijkl} are the elastic-tensor components, e_{ikl} are the piezoelectric-tensor components, ϵ_{ik} are the permittivity-tensor components, γ_{ij} are the components of the temperature-stress tensor, g_i are the pyroelectric coefficients, k_{ij} are the components of the thermal-conduction tensor, c_ε is the heat capacity for constant deformation, ρ is the density, T_0 is the Kelvin temperature in the natural state (all quantities correspond to the isothermal state).

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The surface S bounding the body can be represented as $S = S_u \cup S_\sigma = S_\theta \cup S_q = S_- \cup S_+ \cup S_H$. The boundary conditions are written as

$$\sigma_{ij} n_j|_{S_\sigma} = p_i, \quad u_i|_{S_u} = u_{i0}, \quad q_j n_j|_{S_q} = f, \quad \theta|_{S_\theta} = \theta_0, \quad D_j n_j|_{S_H} = 0, \quad \varphi|_{S_\pm} = \pm\varphi_0, \tag{3}$$

where p_i, u_{i0}, θ_0, and f are known functions, $2\varphi_0$ is the specified potential difference, and n_j are the components of the unit vector of the outer normal to S.

If the quantity φ_0 is unknown, it is determined from the additional condition that the piezoelectric member is connected in the electric circuit:

$$i\omega \int\limits_{S_+} D_n \, dS = -I, \tag{4}$$

where I is the amplitude of the periodic current.

Thus, the behavior of the thermoelectroelastic body is described by a system of five second-order differential equations of complex structure with boundary conditions (3). We construct simpler models of thin-wall pyroelectric members similar to the models of plate and beam bending. To construct such models correctly, one need to use the variational principle of thermoelectroelasticity.

2. Formulation of the Variational Principle. The variational principle of thermoelectroelasticity is formulated in [1], where the equations and boundary conditions are constructed by varying two independent functionals. The variational principle given in the present paper is an analog of the Lagrange variational principle of elastic theory, and the pertinent functional is constructed by the Lagrange multiplier method factors. Let us introduce the concept of a kinematically possible field in the problem of thermoelectroelasticity. We assume that it consists of the functions u_i, φ, and θ which are doubly continuously differentiable in the volume V and satisfy the kinematic boundary conditions $\delta u_i|_{S_u} = 0$, $\delta\varphi|_{S_\varphi} = 0$, and $\delta\theta|_{S_\theta} = 0$. Then, the following statement holds. Among all kinematically possible fields, true fields provide for a steady-state value for the functional L:

$$L = \int\limits_V \Big(\frac{\rho\omega^2}{2} u_i^2 + F_i u_i + \frac{w}{i\omega T_0}\theta - \frac{1}{2} c_{ijkl}\varepsilon_{ij}\varepsilon_{kl} + \epsilon_{ik}\varphi_{,i}\,\varphi_{,k} - \frac{1}{2i\omega T_0} k_{ij}\theta_{,j}\,\theta_{,i}$$

$$+ \frac{c_\varepsilon}{2T_0}\theta^2 - e_{kij}\varepsilon_{ij}\varphi_{,k} + \gamma_{ij}\varepsilon_{ij}\theta - g_i\varphi_{,i}\,\theta\Big) dV + \int\limits_{S_\sigma} p_i u_i \, dS - \frac{1}{i\omega T_0}\int\limits_{S_q} f\theta \, dS. \tag{5}$$

The variational equation $\delta L = 0$ is equivalent to system (1) subject to the constitutive relations (2) and the boundary conditions on S_σ, S_θ, and S_q in (3).

Varying the functional L, we obtain

$$\delta L = \int\limits_V \Big[(\rho\omega^2 u_i + F_i + \sigma_{ij},_j)\delta u_i + D_k,_k \delta\varphi + \Big(\frac{w}{i\omega T_0} + \eta - \frac{1}{i\omega T_0} q_{i,i}\Big)\delta\theta \Big] dV$$

$$+ \int\limits_{S_\sigma} (p_i - \sigma_{ij} n_j)\delta u_i \, dS - \frac{1}{i\omega T_0}\int\limits_{S_q} (f - q_i n_i)\delta\theta \, dS + \int\limits_{S_H} D_k n_k \delta\varphi \, dS = 0. \tag{6}$$

Since the variations of the displacements, potential, and temperature are arbitrary and independent, then, by virtue of the basic lemma of variational calculus, from relation (6) it follows that the factors at relevant variations in both the volume and surface integrals are equal to zero, and, hence, Eq. (1) and the natural boundary conditions in (3).

3. Simple Model for Vibrations of a Thin-Wall Member. We consider the problem of vibrations of a plate of a thermoelectroelastic material of the 6 mm class. We assume that the plate is in a plane strain state and the required fields satisfy the constraints

$$u_2 = 0, \qquad \frac{\partial u_i}{\partial x_2} = \frac{\partial\varphi}{\partial x_2} = \frac{\partial\theta}{\partial x_2} = 0.$$

The section of the plate $x_2 = \text{const}$ is a rectangle Ω $[-l, l] \times [-H, H]$. Vibrations of the plate arise because of the difference in temperature between the boundaries $x_3 = \pm H$:

$$\sigma_{13}|_{x_3=\pm H} = \sigma_{33}|_{x_3=\pm H} = 0, \qquad \varphi|_{x_3=\pm H} = \pm\varphi_0, \qquad \theta|_{x_3=\pm H} = \theta_\pm,$$

$$\sigma_{13}|_{x_1=\pm l} = \sigma_{11}|_{x_1=\pm l} = 0, \qquad D_1|_{x_1=\pm l} = 0, \qquad q_1|_{x_1=\pm l} = 0.$$

In this formulation, the constants $\theta_\pm$ are known and the unknown potential φ_0 is determined from the additional condition (4), which, in this case, becomes

$$\int_{-l}^{l} D_3(x_1, H)\, dx_1 = 0. \tag{7}$$

Let us construct a simplified model for the deformation of the plate with $\varepsilon = H/l \ll 1$ using the classical Kirchhoff hypotheses and assuming

$$u_1 = u_{10}(x_1) - x_3 u'_{30}(x_1), \quad u_3 = u_{30}(x_1), \quad \varphi = (x_3/H)\varphi_0, \quad \theta = T_1 + (x_3/H)T_2, \tag{8}$$

where $T_1 = (\theta_+ + \theta_-)/2$ and $T_2 = (\theta_+ - \theta_-)/2$. Substituting (8) into the functional, we obtain its variation. Setting the factors at the independent variations δu_{10} and δu_{30} equal to zero, we obtain a system of equations and boundary conditions that describe vibrations of a thin-wall member within the framework of the thermoelectroelasticity model.

The obtained system of equations and boundary conditions is naturally divided into two independent problems, which can conditionally be called the problems of "tension–compression" (Problem 1) and "bending" (Problem 2) of a thin-wall member.

Problem 1. The unknown function u_{10} satisfies the equation and the boundary conditions

$$c^*_{11} u''_{10} + \rho\omega^2 u_{10} = 0, \qquad c^*_{11} u'_{10}|_{x_1=\pm l} = \gamma^*_{11} T_1 - (e^*_{31}/H)\varphi_0.$$

Problem 2. The unknown function u_{30} satisfies the equation and the boundary conditions

$$c^*_{11} u^{(4)}_{30} + \rho\omega^2 u''_{30} - \frac{3\rho\omega^2}{H^2}\, u_{30} = 0, \quad (c^*_{11} u'''_{30} + \rho\omega^2 u'_{30})|_{x_1=\pm l} = 0, \quad c^*_{11} u''_{30}|_{x_1=\pm l} = -\frac{\gamma^*_{11}}{H}\, T_2.$$

Here $c^*_{11} = c_{11} - c^2_{13}/c_{33}$, $\gamma^*_{11} = \gamma_{11} - c_{13}\gamma_{33}/c_{33}$, and $e^*_{31} = e_{31} - e_{33}c_{13}/c_{33}$.

It should be noted that the difference in temperature between the boundaries enters into the equation of Problem 2, and the sum enters into the equation of Problem 1. In Problem 2, the equation and the first boundary condition differ from the equation and boundary condition of the classical problem of beam bending only in the terms $\rho\omega^2 u''_{30}$ and $\rho\omega^2 u'_{30}$, respectively, which take into account rotary inertia [6]. The term $-(\gamma^*_{11}/H)T_2$, which depends on the difference of the temperatures θ_+ and θ_-, plays the role of the bending moment applied at the edges $x_1 = \pm l$.

The additional condition (7) has the form

$$\int_{-l}^{l} \Big(e^*_{31}(u'_{10} - H u''_{30}) - \frac{\epsilon^*_{33}}{H}\, \varphi_0 + g^*_3(T_1 + T_2) \Big) dx_1 = 0, \tag{9}$$

where $\epsilon^*_{33} = \epsilon_{33} + e^2_{33}/c_{33}$ and $g^*_3 = g_3 + \gamma_{33}e_{33}/c_{33}$.

The solutions of Problems 1 and 2 are written as

$$u_{10} = \frac{l(\gamma^*_{11}T_1 - (e^*_{31}/H)\varphi_0)}{c^*_{11} k \cos k}\, \sin\Big(\frac{k}{l}\, x_1\Big), \quad u_{30} = \frac{\gamma^*_{11}}{c^*_{11} H K_0}\, (f(\xi_2)\cosh\,(\xi_1 x_1) - f(\xi_1)\cosh\,(\xi_2 x_1))T_2,$$

where $k = \omega l/c$, $c = \sqrt{c^*_{11}/\rho}$, $\varepsilon = H/l$, $\xi_{1,2} = (k/(\sqrt{2}l))\sqrt{-1 \pm \sqrt{1 + 12/(k^2\varepsilon^2)}}$, $f(\xi_1) = \xi_1(\xi_1^2 + k^2/l^2)\sinh\,(\xi_1 l)$, and $K_0 = \xi_2^2 f(\xi_1)\cosh\,(\xi_2 l) - \xi_1^2 f(\xi_2)\cosh\,(\xi_1 l)$.

The induced potential is determined from (9) as follows:

$$\varphi_0 = \varphi_1 T_1 + \varphi_2 T_2,$$

$$\varphi_1 = H(e^*_{31}\gamma^*_{11}\sin k + g^*_3 c^*_{11} k \cos k)/d_0, \qquad d_0 = \epsilon^*_{33} c^*_{11} k \cos k + e^{*2}_{31}\sin k,$$

$$\varphi_2 = c^*_{11} k \cos k (g^*_3 l - (e^*_{31}\gamma^*_{11}/K_0)(f(\xi_2)\xi_1 \sinh\,(\xi_1 l) - f(\xi_1)\xi_2 \sinh\,(\xi_2 l)))/d_0.$$

Let us analyze the obtained formulas for calculating the induced potential. In a pyroelectric member there are two sets of resonant frequencies that correspond to longitudinal resonances, determined from the condition $d_0(k_j) = 0$, and bending resonances, determined from the equation $K_0(k_i) = 0$. At these frequencies, the induced potential tends to infinity. It should be noted that at $T_2 = 0$ there is no bending.

4. Improved Model of Vibrations of a Thin-Wall Member. The simplest model of deformation of a thin-wall member studied in Sec. 3 ignores the temperature distribution along the x_1 coordinate and yields an infinite increase of the potential at the frequencies of the longitudinal and bending resonances.

We consider an improved model of vibrations of a thin-wall member, assuming that

$$u_1 = u_{10}(x_1) - x_3(e_{15}/c_{44})\Phi_0'(x_1) - x_3 u_{30}'(x_1), \qquad u_3 = u_{30}(x_1),$$
$$\varphi = (1 - x_3^2/H^2)\Phi_0(x_1) + x_3\varphi_0/H, \qquad \theta = (1 - x_3^2/H^2)\theta_0(x_1) + (x_3/H)T_2 + (x_3^2/H^2)T_1. \tag{10}$$

In this case, the following kinematic boundary conditions are satisfied: $\varphi(x_1, \pm H) = \pm\varphi_0$ and $\theta(x_1, \pm H) = \theta_\pm$. In addition, $\sigma_{13}(x_1, \pm H) = 0$, and the additionally introduced functions $\Phi_0(x_1)$ and $\theta_0(x_1)$ have a simple physical meaning: $\varphi(x_1, 0) = \Phi_0(x_1)$ and $\theta(x_1, 0) = \theta_0(x_1)$.

Varying the functional (5) with allowance for (10) and setting the factors at the independent variations δu_{10}, δu_{30}, $\delta\Phi_0$, and $\delta\theta_0$ equal to zero, we obtain a system equations and boundary conditions, which is also divided in a natural fashion into two independent problems.

Problem 1a (generalized problem of tension–compression)**.** The unknown functions u_{10} and θ_0 satisfy the following equations and boundary conditions:

$$c_{11}^* u_{10}'' + \rho\omega^2 u_{10} - (2/3)\gamma_{11}^*\theta_0' = 0,$$

$$\gamma_{11}^* u_{10}' + \frac{4}{5}\,\frac{k_{11}}{i\omega T_0}\,\theta_0'' - \Big(\frac{2}{H^2}\,\frac{k_{33}}{i\omega T_0} - \frac{4}{5}\,\frac{c_\varepsilon^*}{T_0}\Big)\theta_0 = -\Big(\frac{2}{H^2}\,\frac{k_{33}}{i\omega T_0} + \frac{1}{5}\,\frac{c_\varepsilon^*}{T_0}\Big)T_1 + \frac{g_3^*}{H}\,\varphi_0,$$

$$(c_{11}^* u_{10}' - (2/3)\gamma_{11}^*\theta_0)|_{x_1=\pm l} = \gamma_{11}^* T_1/3 - (e_{31}^*/H)\varphi_0, \qquad \theta_0'|_{x_1=\pm l} = 0.$$

Problem 2a (generalized bending problem)**.** The unknown functions u_{30} and Φ_0 satisfy the following equations and boundary conditions

$$c_{11}^* u_{30}^{(4)} + \rho\omega^2 u_{30}'' - \frac{3\rho\omega^2}{H^2}\,u_{30} + c_{11}^*\,\frac{e_{15}}{c_{44}}\,\Phi_0^{(4)} + \Big(\rho\omega^2\,\frac{e_{15}}{c_{44}} + \frac{2e_{31}^*}{H^2}\Big)\Phi_0'' = 0,$$

$$c_{11}^*\,\frac{e_{15}}{c_{44}}\,u_{30}^{(4)} + \Big(\rho\omega^2\,\frac{e_{15}}{c_{44}} + \frac{2e_{31}^*}{H^2}\Big)u_{30}'' + c_{11}^*\,\frac{e_{15}^2}{c_{44}^2}\,\Phi_0^{(4)}$$
$$+ \Big(\rho\omega^2\,\frac{e_{15}^2}{c_{44}^2} + \frac{4e_{31}^*}{H^2}\,\frac{e_{15}}{c_{44}} + \frac{19}{H^2}\,\frac{e_{15}^2}{c_{44}} + \frac{8}{5}\,\frac{\epsilon_{11}}{H^2}\Big)\Phi_0'' - \frac{4\epsilon_{33}^*}{H^4}\,\Phi_0 = \frac{2g_3^*}{H^3}\,T_2,$$

$$\Big(c_{11}^* u_{30}''' + \rho\omega^2 u_{30}' + c_{11}^*\,\frac{e_{15}}{c_{44}}\,\Phi_0''' + \Big(\rho\omega^2\,\frac{e_{15}}{c_{44}} + \frac{2e_{31}^*}{H^2}\Big)\Phi_0'\Big)\Big|_{x_1=\pm l} = 0,$$

$$\Big(c_{11}^*\,\frac{e_{15}}{c_{44}}\,u_{30}''' + \rho\omega^2\,\frac{e_{15}}{c_{44}}\,u_{30}' + c_{11}^*\,\frac{e_{15}^2}{c_{44}^2}\,\Phi_0''' + \Big(\rho\omega^2\,\frac{e_{15}^2}{c_{44}^2} + \frac{2e_{31}^*}{H^2}\,\frac{e_{15}}{c_{44}} + \frac{19}{H^2}\,\frac{e_{15}^2}{c_{44}} + \frac{8}{5}\,\frac{\epsilon_{11}}{H^2}\Big)\Phi_0'\Big)\Big|_{x_1=\pm l} = 0,$$

$$\Big(c_{11}^* u_{30}'' + c_{11}^*\,\frac{e_{15}}{c_{44}}\,\Phi_0'' + \frac{2e_{31}^*}{H^2}\,\Phi_0\Big)\Big|_{x_1=\pm l} = -\frac{\gamma_{11}^*}{H}\,T_2.$$

The additional condition (7) has the form

$$\int\limits_{-l}^{l}\Big(e_{31}^*\Big(u_{10}' - H\,\frac{e_{15}}{c_{44}}\,\Phi_0'' - Hu_{30}''\Big) - \frac{\epsilon_{33}^*}{H}(\varphi_0 - 2\Phi_0) + g_3^*(T_1 + T_2)\Big)\,dx_1 = 0. \tag{11}$$

It should be noted that the sum of the temperatures on the boundaries enters into Problem 1a, and their difference enters into Problem 2a. Problems 1a and 2a are much more complicated than Problems 1 and 2. Their solutions are expressed in terms of roots of biquadratic and bicubic characteristic equation.

The induced potential has the same structure as in Sec. 3: $\varphi_0 = \varphi_1 T_1 + \varphi_2 T_2$, and φ_1 and φ_2 are found from condition (11).

We construct a solution of Problem 1a for the case where $T_2 = 0$, i.e., bending is absent and $\Phi_0 = 0$, $u_{30} = 0$. In this case, the solution of Problem 1 has the form

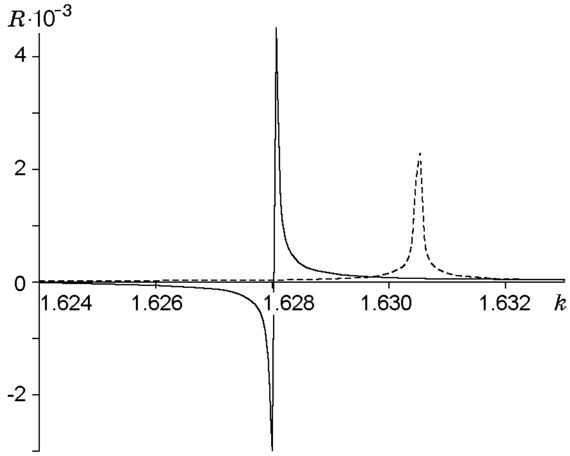

Fig. 1

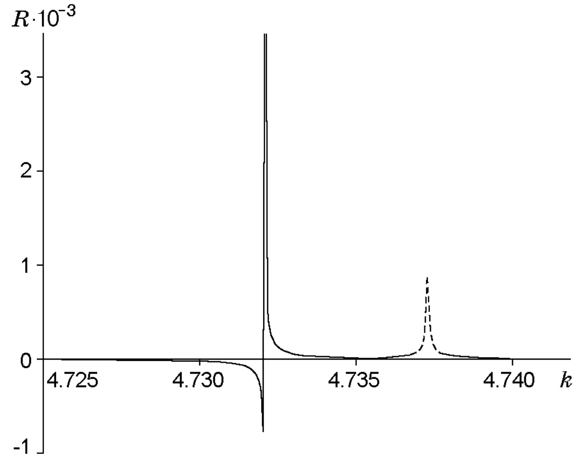

Fig. 2

$$u_{10} = \sum_{i=1}^{2} A_i \sinh\,(\lambda_i x_1), \qquad \theta_0 = \sum_{i=1}^{2} B_i \cosh\,(\lambda_i x_1) + L_1 T_1 - \frac{L_2}{H}\,\varphi_0,$$

$$A_i = (2/3)(\gamma_{11}^*/c_{11}^*)\lambda_i X_i, \quad B_i = (\lambda_i^2 + k^2/l^2)X_i, \quad L_1 = b_3/b_2, \quad L_2 = b_4/b_2.$$

Here λ_i are solutions of the characteristic equation

$$(4/5)\lambda^4 + (2\gamma_{11}^{*2} b_1/(3c_{11}^*) - b_2 + 4k^2/(5l^2))\lambda^2 - b_2 k^2/l^2 = 0,$$

where $b_1 = T_0 ikc/(k_{11}l)$, $b_2 = (10lk_{33} - 4c_\varepsilon^* k_{11} H^2 ikc)/(5H^2 k_{11} l)$, $b_3 = (10lk_{33} + c_\varepsilon^* k_{11} H^2 ikc)/\ (5H^2 k_{11} l)$, and $b_4 = g_3^* T_0 ikc/(k_{11}l)$, and X_i are obtained from boundary conditions

$$X_1 = \frac{3}{2}\,\frac{c_{11}^*}{\gamma_{11}^*}\,\frac{l^2}{k^2}\,\frac{F(\lambda_2)}{D_0}\Big(N_1 T_1 - N_2\,\frac{\varphi_0}{H}\Big), \qquad X_2 = -\frac{3}{2}\,\frac{c_{11}^*}{\gamma_{11}^*}\,\frac{l^2}{k^2}\,\frac{F(\lambda_1)}{D_0}\Big(N_1 T_1 - N_2\,\frac{\varphi_0}{H}\Big),$$

where

$F(\lambda_1) = \lambda_1(\lambda_1^2 + k^2/l^2)\sinh\,(\lambda_1 l)$, $N_1 = \gamma_{11}^*(1 + 2b_3/b_2)/(3c_{11}^*)$, $N_2 = (e_{31}^* + 2\gamma_{11}^* b_4/(3b_2))/c_{11}^*$, and $D_0 = F(\lambda_1)\cosh\,(\lambda_2 l) - F(\lambda_2)\cosh\,(\lambda_1 l)$.

The potential φ_0 is determined from the additional condition (11):

$$\varphi_0 = T_1 H\,\frac{g_3^* lk^2 D_0 + \lambda_1\lambda_2 l^2 e_{31}^* N_1 \sinh\,(\lambda_1 l)\sinh\,(\lambda_2 l)(\lambda_2^2 - \lambda_1^2)}{\epsilon_{33}^* lk^2 D_0 + \lambda_1\lambda_2 l^2 e_{31}^* N_2 \sinh\,(\lambda_1 l)\sinh\,(\lambda_2 l)(\lambda_2^2 - \lambda_1^2)}.$$

In this case, the value of the potential φ_0 is complex. Then, separating the real part, we obtain $\varphi_0 = AR\sin\,(\omega t + \delta_3)$, where A is a dimensional factor and R is the dimensionless amplitude which depends on k.

Figures 1 and 2 gives curves of the potential amplitude versus the dimensionless frequency k for a pyroelectric member made of barium titanate (the solid curves correspond to the simplest model and the dashed curves correspond to the improved model). In Fig. 1, the curves are shown in the neighborhood of the first longitudinal resonance and in Fig. 2, they are shown in the neighborhood of the second longitudinal resonance. A comparison of these curves show that in the improved model, the amplitudes are finite and shifted toward increasing k. An analysis shows that the improved model can be used to describe coupled vibrations of pyroelectric members.

In the case of $\gamma_{11}^* = 0$ at $T_2 = 0$, which corresponds to the absence of coupling, in Problems 1 and 1a, the solutions for the induced potential in both models coincide and are brought to the form $\varphi_1 = T_1 H g_3^* c_{11}^* k \cos k/d_0$.

In modeling vibrations of pyroelectric members, it is common to use the concept of weak coupling, in which the quantities γ_{11}^* and g_3^* are considered small and regular expansions in these parameters are constructed. It is easy to show that with this approach, infinite jumps of the potential amplitude at the frequencies of the longitudinal and transverse resonant are retained. In the nonresonant fields, this approach yields results close to those obtained for the improved model.

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